3　関数の補間／補遺

【練習問題 3-1】直接解法
求める式を $f(x) = a_0 + a_1x + a_2x^2$ とおいて，３点を代入すると．

$$\begin{aligned} a_0 + a_1 + a_2 &= 2 \\ a_0 + 2a_1 + 4a_2 &= 3 \\ a_0 + 3a_1 + 9a_2 &= 6 \end{aligned}$$

この連立１次方程式を解くと $a_0 = 3, a_1 = -2, a_2 = 1$ となり，求める式は $f(x) = 3 - 2x + x^2$ となる．

【練習問題 3-2】Lagrange 補間公式

$$\begin{aligned} P(x) &= \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}f(x_0) + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}f(x_1) + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}f(x_2) \\ &= \frac{(x-2)(x-3)}{(1-2)(1-3)}2 + \frac{(x-1)(x-3)}{(2-1)(2-3)}3 + \frac{(x-1)(x-2)}{(3-1)(1-2)}6 \\ &= x^2 - 2x + 3 \end{aligned}$$

【練習問題 3-3】Newton の補間公式
差分商を求めてみる．

$$\begin{aligned} f_1[x_0, x_1] &= \frac{f(x_0) - f(x_1)}{x_0 - x_1} = \frac{2-3}{1-2} = 1 \\ f_1[x_1, x_2] &= \frac{f(x_1) - f(x_2)}{x_1 - x_2} = \frac{3-6}{2-3} = 3 \\ f_2[x_0, x_2] &= \frac{f_1[x_0, x_1] - f_1[x_1, x_2]}{x_0 - x_2} = \frac{1-3}{1-3} = 1 \end{aligned}$$

この結果を用いて

$$\begin{aligned} P(x) &= f(x_0) + (x-x_0)f_1[x_0, x_1] + (x-x_0)(x-x_1)f_2[x_0, x_2] \\ &= 2 + (x-1)1 + (x-1)(x-2)1 \\ &= 3 - 2x + x^2 \end{aligned}$$

【練習問題 3-4】Newton の定差前進補間公式
公式に２次多項式で当てはめると，$h = 1, s = x - x_0$ より

$$\begin{aligned} P(x) &= f_0 + s\Delta f_0 + \frac{s(s-1)}{2!}\Delta^2 f_0 \\ &= f(x_0) + (x-x_0)(f(x_1) - f(x_0)) + (x-x_0)(x-x_0-1)((f(x_2) - f(x_1)) - (f(x_1) - f(x_0)))\frac{1}{2!} \\ &= f(1) + (x-1)(f(2) - f(1)) + \frac{1}{2}(x-1)(x-2)((f(3) - f(2)) - (f(2) - f(1)) \\ &= 2 + (x-1)(3-2) + \frac{1}{2}(x-1)(x-2)((6-3) - (3-2)) \\ &= 3 - 2x + x^2 \end{aligned}$$

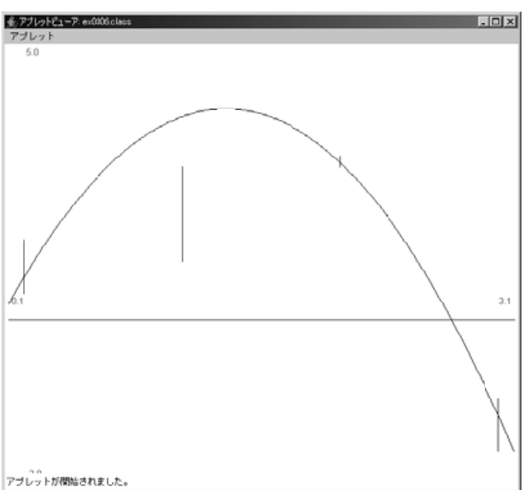
【練習問題 3-5】最小二乗法

$$\begin{pmatrix} A_{0,0} & A_{0,1} & A_{0,2} \\ A_{1,0} & A_{1,1} & A_{1,2} \\ A_{2,0} & A_{1,2} & A_{2,2} \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} b_0 \\ b_1 \\ b_2 \end{pmatrix} \tag{1}$$

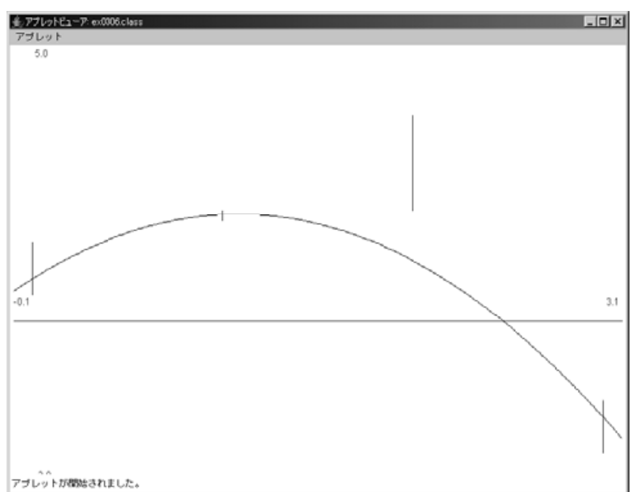
$$\begin{aligned}
A_{0,0} &= \sum_{i=1}^{N} w_i x_i^0 = w_0 + w_1 + w_2 + w_3 = 109.2 \\
A_{0,1} = A_{1,0} &= \sum_{i=1}^{N} w_i x_i^1 = w_0 x_0 + w_1 x_1 + w_2 x_2 + w_3 x_3 = 213.2 \\
A_{1,1} = A_{0,2} = A_{2,0} &= \sum_{i=1}^{N} w_i x_i^2 = w_0 x_0^2 + w_1 x_1^2 + w_2 x_2^2 + w_3 x_3^2 = 437.2 \\
A_{2,1} = A_{1,2} &= \sum_{i=1}^{N} w_i x_i^3 = w_0 x_0^3 + w_1 x_1^3 + w_2 x_2^3 + w_3 x_3^3 = 909.2 \\
A_{2,2} &= \sum_{i=1}^{N} w_i x_i^4 = w_0 x_0^4 + w_1 x_1^4 + w_2 x_2^4 + w_3 x_3^4 = 1,925.2 \\
b_0 &= \sum_{i=1}^{N} w_i f_i x_i^0 = w_0 f_0 + w_1 f_1 + w_2 f_2 + w_3 f_3 = 298.5 \\
b_1 &= \sum_{i=1}^{N} w_i f_i x_i^1 = w_0 f_0 x_0 + w_1 f_1 x_1 + w_2 f_2 x_2 + w_3 f_3 x_3 = 578.5 \\
b_2 &= \sum_{i=1}^{N} w_i f_i x_i^2 = w_0 f_0 x_0^2 + w_1 f_1 x_1^2 + w_2 f_2 x_2^2 + w_3 f_3 x_3^2 = 1,130.5
\end{aligned}$$

これを解くと

$$F(x) = 0.80964 + 4.99746x - 1.95685x^2$$



kai^2 = 4.57



2番目の点と３番目の点の重みを交換

kai^2 = 4.57